

## B. Tech.

### Second SEMESTER EXAMINATION, 2008-09

## Special Carry over Examination Engineering Physics-II

Note : This question paper contains three sections.

#### Section - A

Q. 1. Attempt all parts. All parts carry equal marks.

(a) If the wavelength associated with a particle A is  $5 \times 10^{-8} \text{ m}$  whose momentum is twice that of another particle B, the wavelength associated with particle B will be

Ans.  $10^{-7}$

(b) In Compton Effect, a photon can be scattered in any direction while the electron can recoil only in the onward direction at angles less than

Ans.  $90^\circ$

Pick the correct choice from following :

(c) The hysteresis loop of a ferro electric material at Curie temperature, merges in to a

- (i) Point                      (ii) Circle
- (iii) Straight line        (iv) Hyperbola

Ans. Straight line

(d) The relation between dipole moment  $P$ , polarizability  $\alpha$  and electric field  $E$  is

- (i)  $p = \frac{\alpha}{E}$                       (ii)  $E = p\alpha$
- (iii)  $p = \alpha E$                 (iv)  $p = \alpha E^2$

Ans.  $p = \alpha E$

(e) At high frequency, generally which polarization is not accountable

- (i) Electronic              (ii) Ionic
- (iii) Orientational
- (iv) Electronic and Ionic

Ans. Orientational

(f) The susceptibility of a diamagnetic substance

- (i) increases with temperature
- (ii) decreases with temperature
- (iii) does not vary with temperature
- (iv) first decreases and then increases with temperature

Ans. decreases with temperature

(g) Ultrasonic waves are

- (i) Transverse waves
- (ii) Longitudinal waves
- (iii) Stationary waves
- (iv) Electromagnetic waves

Ans. Longitudinal waves

(h) The Poynting vector describes the flow of

- (i) Energy per unit area unit time
- (ii) Energy per unit area
- (iii) Energy per unit volume
- (iv) Energy per unit volume per unit time

Ans. (ii) Energy per unit area

(i) The temperature at which a conductor becomes a superconductor is called

- (i) Superconducting temperature
- (ii) critical temperature
- (iii) Onne's temperature
- (iv) Transition temperature

Ans. (ii) critical temperature

(j) The structure of Buckyball  $C_{60}$  molecule consists of

- (i) 12 hexagonal and 20 pentagonal faces
- (ii) 20 hexagonal and 12 pentagonal faces
- (iii) 24 hexagonal and 15 pentagonal faces
- (iv) 15 hexagonal and 24 pentagonal faces

Ans. (ii) 20 hexagonal and 12 pentagonal faces

### Section B

**Q.2 Attempt any three parts**

**All parts carry equal marks**

**(a) Calculate the kinetic energy of an electron of its de-Broglie wavelength equals the wavelength of the yellow line of sodium (5796 Å)**

Ans. Given  $\lambda = 5896 \text{ Å} = 5896 \times 10^{-10} \text{ m}$

we know that,  $\lambda = \frac{h}{mv}$

$$v = \frac{h}{\lambda m} = \frac{6.63 \times 10^{-34}}{(5896 \times 10^{-10}) \times (9.1 \times 10^{-31})}$$

$$= 0.1236 \times 10^4$$

$$= 1236 \times 10^2 \text{ m/s.}$$

$$\text{K.E.} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (1236 \times 10^2)^2$$

$$= 69493 \times 10^{-27} \text{ J}$$

$$= \frac{69493 \times 10^{-27}}{1.6 \times 10^{-19}}$$

$$= 434 \times 10^{-6} \text{ e.v.}$$

**(b) An electron is confined to a box of length  $1.1 \text{ Å}$ . Calculate the minimum uncertainty in its velocity.**

Ans. Given:  $\Delta x = 1.1 \text{ Å} = 1.1 \times 10^{-10} \text{ m}$

From uncertainty principle,

$$\Delta x \Delta p = \frac{h}{2\pi}$$

$$\text{or } \Delta n \times m \times \Delta v = \frac{h}{2\pi}$$

$$\text{or } \Delta v = \frac{h}{2\pi m \times \Delta x}$$

$$= \frac{6.63 \times 10^{-34}}{(2 \times 3.14) \times (9.1 \times 10^{-31}) \times (1.1 \times 10^{-10})}$$

$$= 0.1055 \times 10^7$$

$$= 1.06 \times 10^6 \text{ m/s}$$

**(c) The solar energy that the earth receives on its surface is  $1.365 \text{ watt/m}^2$ . Calculate the amplitudes of electric and magnetic fields at the earth surface.**

Ans. Given  $S = EH = 1365 \text{ watt/m}^2 \dots (1)$

also we know that

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.72 \dots (2)$$

Multiplying equation (1) by (2) we have

$$E = \sqrt{1365 \times 376.72} = 717.1 \text{ volt/m}$$

Dividing eq (1) by (2)

$$H = \sqrt{\frac{1365}{376.72}}$$

$$= 1.9 \text{ amp-turn/m.}$$

Thus amplitude of electric and magnetic fields of radiation are,

$$E_0 = E\sqrt{2} = 717.1 \times \sqrt{2} = 101398 \text{ volt/m.}$$

$$\text{and } H_0 = H\sqrt{2} = 1.9 \times \sqrt{2} = 2.69 \text{ amp-turn/m.}$$

**(d) Calculate the electric polarizability of an argon atom whose dielectric constant  $\epsilon_r$  is 1.0024 at NTP and number of atoms in unit volume of argon gas are  $2.7 \times 10^{25} \text{ atoms/m}^3$**

Ans. Given:  $\epsilon_r = 1.0024$

$$N = 2.7 \times 10^{25} \text{ atoms/m}^3$$

we know that, the electronic polarizability is,

$$\alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

$$= \frac{(8.85 \times 10^{-12}) (1.0024 - 1)}{2.7 \times 10^{25}}$$

$$= 7867 \times 10^{-41} \text{ F-m}^2$$

(e) Calculate the temperature at which the critical magnetic field is two-third of the value at 0°K for a tin superconductor with critical temperature 4°K

Ans. Given :  $T = 4\text{K}$

$$\frac{H_c(T)}{H_c(0)} = \frac{2}{3}$$

We know that

$$T_c = \frac{T}{\left[1 - \frac{H_c(T)}{H_c(0)}\right]^{1/2}}$$

$$= \frac{4}{\left[1 - \frac{2}{3}\right]^{1/2}} = \frac{4}{(1/3)^{1/2}}$$

$$= 4\sqrt{3} = 6.93 \text{ K}$$

**Section : C**

Attempt all questions. All questions carry equal marks :

Q.3. Attempt any one part of the following :

(a) Establish a relation between group velocity  $V_g$  and phase velocity  $V_p$  of a wave packet in a dispersive medium and show that these velocities are equal in non dispersive medium.

Ans (a) we know that group velocity

$$v_g = \frac{d\omega}{dk} \quad \dots (1)$$

and phase velocity,  $v_p = \frac{\omega}{k}$

$$\text{or} \quad \omega = k \cdot v_p \quad \dots (2)$$

From equation (2), equation (1) becomes,

$$\therefore v_g = \frac{d}{dk} (k v_p)$$

$$v_p + k \cdot \frac{dv_p}{dk}$$

$$= v_p + \frac{2\pi}{\lambda} \frac{dv_p}{d\left(\frac{2\pi}{\lambda}\right)} \quad \left(\because k = \frac{2\pi}{\lambda}\right)$$

$$= v_p - \lambda \frac{dv_p}{d\lambda}$$

But for non dispersive medium,

$$\frac{dv_p}{d\lambda} = 0$$

$\therefore v_g = v_p$  Hence proved.

(b) Obtain an expression for the energy states of a particle in one dimensional box.

Ans. The Particle in one Dimensional-Box

Let us consider the particle restricted to move along the  $x$ -axis between the limits  $x=0$  to  $x=L$ .

Let the potential energy  $V$  of the particle, inside the box is zero but this potential rises upto infinite on the outside.

i.e.,  $V=0$ ,  $\forall$  such that  $0 \leq x \leq L$

$V=\infty$ ,  $\forall$  such that  $x < 0$  and  $x > L$  ... (1)

The Schrodinger wave equation for a particle in the box will be as follows

$$\nabla^2 \Psi + \frac{2m}{\hbar^2} (E - V) \Psi = 0$$

$$\frac{d^2 \Psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} E \Psi = 0$$

[Since the particle in one-dimensional box

$$\therefore \frac{d^2 \Psi}{dx^2} = -\frac{\partial^2 \Psi}{\partial x^2} \quad \dots (2)$$

$\therefore y$  and  $z$  are zero]

Schrodinger wave eqn. outside the box can not be specified.

$$\text{Let us consider, } k^2 = \frac{8\pi^2 m E}{\hbar^2} \quad \dots (3a)$$

$$\therefore \frac{d^2 \Psi}{dx^2} + k^2 \Psi = 0 \quad \dots (3b)$$

the auxiliary equ. will be,

$$m^2 + k^2 = 0$$

$$m = k$$

∴ Sol. of diff. equ. is,

$$\psi = A \cos kx + B \sin kx \quad \dots(4)$$

where A and B are arbitrary constants and can be determined by boundary values.

As the particle cannot have infinite energy, it cannot exist outside the box.  $\psi$  must be zero outside the walls and also zero at the walls.

$$\therefore \text{at } x = 0, \psi(x) = 0$$

$$\text{from eq. (4)} \quad 0 = A \cos 0 + B \sin 0$$

$$A = 0$$

$$\text{but at, } n = L, \psi = 0$$

$$\text{Thus } 0 = 0 \cdot \cos kL + B \sin(kL)$$

$B \neq 0$  (since both arbitrary constants cannot be zero)

$$\therefore \sin kL = 0$$

$$\sin kL = \sin(0)$$

$$kL = n\pi + (-1)^n \cdot 0$$

$$k = \frac{n\pi}{L}$$

$$\text{equation 3(a) we have } k^2 = \frac{8\pi^2 m E}{h^2}$$

Thus energy,

$$\begin{aligned} E &= \frac{k^2}{8\pi^2 m} = \frac{\frac{n^2 \pi^2}{L^2} \cdot h^2}{8\pi^2 m} \\ &= E_n = \frac{n^2 h^2}{8mL^2} \quad \dots(5) \end{aligned}$$

where  $n = 1, 2, 3, \dots$

The relation 5th gives arbitrary, Eigen values  $E_n$  which represents the energy of  $n$ th level.

**Q.4. Attempt any one part of the following :**

**(a) What do you understand by Compton shift? Show that it is independent of the wavelength of the incident photon.**

**Ans.** When a chromatic beam of high frequency radiation. For example x-rays is scattered by a substance, the scattered radiation contains the radiation of lower frequency or greater wavelength along a radiation of unchange wavelength along a radiation of unchange wavelength in the scattered light is called unmodified radiation while the radiation of greater wave length is called modified radiation. The phenomena is called the Compton effect & the change in  $\Delta\lambda$  is called the Compton shift.

Loss in photon energy = Gain in electroenergy

$$h\nu - h\nu' = K \quad \dots(1)$$

Since, the collision is elastic therefore, total linear momentum of photon & electron before collision is equal to after collision

**Along x-axis**

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos \phi + P \cos \theta$$

$$P \cos \theta = h\nu - h\nu' \cos \phi \quad \dots(2)$$

**Along y-axis**

$$0 + 0 = \frac{h\nu'}{c} \sin \phi - P \sin \theta$$

$$P \sin \theta = h\nu' \sin \phi \quad \dots(3)$$

Squaring & adding equation (2) & (3)

$$\begin{aligned} P^2 c^2 (\sin^2 \theta + \cos^2 \theta) &= (h\nu - h\nu' \cos \phi)^2 + (h\nu' \sin \phi)^2 \\ P^2 c^2 &= h^2 \nu^2 - 2h^2 \nu \nu' \cos \phi + h^2 \nu'^2 \quad \dots(4) \end{aligned}$$

Also, Initial energy of  $e^-$  + Gain in energy = Final energy of electron

$$m_0 c^2 + K = \sqrt{(m_0 c^2)^2 + P^2 c^2}$$

Now, squaring both sides

$$m_0^2 c^4 + K^2 + 2m_0 c^2 K = m_0^2 c^4 + P^2 c^2$$

Now, comparing it with equation (4)

$$\begin{aligned} K^2 + 2m_0 c^2 K &= h^2 \nu^2 - 2h^2 \nu \nu' \cos \phi + h^2 \nu'^2 \end{aligned}$$

$$\begin{aligned}
 \therefore K &= hv - hv' \\
 (hv - hv')^2 + 2m_0c^2(hv - hv') &= h^2v^2 - 2h^2vv' \cos \phi + h^2v'^2 \\
 -2h^2vv' + 2m_0c^2h(v - v') &= 2h^2vv' \cos \phi
 \end{aligned}$$

Now, dividing by  $2h^2c^2$ .

$$-\frac{vv'}{c^2} + \frac{m_0}{h}(v - v') = \frac{-vv'}{c^2} \cos \phi$$

$$\frac{m_0}{h}(v - v') = \frac{vv'}{c^2} (1 - \cos \phi)$$

$$\frac{m_0c}{h} \left( \frac{v}{c} - \frac{v'}{c} \right) = \frac{v}{c} \cdot \frac{v'}{c} (1 - \cos \phi)$$

$$\therefore \frac{v}{c} = \frac{1}{\lambda} \text{ \& \ } \frac{v'}{c} = \frac{1}{\lambda'}$$

$$\frac{m_0c}{h} \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) = \frac{1}{\lambda\lambda'} (1 - \cos \phi) \quad \dots (5)$$

$$\frac{m_0c}{h} (\lambda' - \lambda) = 1 - \cos \phi$$

$$\lambda' - \lambda = \frac{\lambda}{m_0c} (1 - \cos \phi)$$

Now for Direction of the Recoil Electron

Dividing eq. (3) by (2)

$$\tan \theta = \frac{hv' \sin \phi}{hv - hv' \cos \phi} = \frac{v' \sin \phi}{v - v' \cos \phi} \quad \dots (6)$$

From eq. (5), we have

$$\frac{1}{v'} = \frac{1}{v} + \frac{h}{m_0c^2} (1 - \cos \phi)$$

$$\frac{u'}{v} = 1 + \frac{h}{m_0c^2} 2 \sin^2 \phi/2$$

$$v' = \frac{1}{1 + \left( \frac{hv}{m_0c^2} \right) \times 2 \sin^2 \phi/2}$$

Putting the value of  $v'$  in (6)

$$\tan \theta = \frac{v \sin \phi / (1 + \alpha 2 \sin^2 \phi/2)}{\left[ \frac{1}{1 + \alpha 2 \sin^2 \phi/2} \right] \cos \phi}$$

$$\text{where } \alpha = \frac{hv}{m_0c^2}$$

$$\begin{aligned}
 \tan \theta &= \frac{\sin \phi}{1 + 2\alpha \sin^2 \phi/2 - \cos \phi} \\
 &= \frac{2 \sin \phi/2 \cos \phi/2}{1 + 2\alpha \sin^2 \phi/2 - \cos \phi} \\
 &= \frac{2 \sin \phi/2 \cos \phi/2}{(1 - \cos \phi) + 2\alpha \sin^2 \phi/2} \\
 &= \frac{2 \sin \phi/2 \cos \phi/2}{2 \sin^2 \phi/2 + 2\alpha \sin^2 \phi/2} \\
 &= \frac{2 \sin \phi/2 \cos \phi/2}{2 \sin^2 \phi/2 (1 + \alpha)} \\
 &= \frac{\cot \phi/2}{1 + \alpha} \\
 \tan \theta &= \frac{\cot \phi/2}{\left( \frac{1 + hv}{m_0c^2} \right)}
 \end{aligned}$$

Which gives the direction of the recoil electron in terms of the frequency of the incident photon & the direction of the scattered photon  
Now at  $\phi = \frac{\pi}{2} (90^\circ)$

$$E = \frac{\frac{h^2 v^2}{m_0c^2}}{1 + \frac{hv}{m_0c^2}}$$

$$\text{Let us put } a = \frac{hv}{m_0c^2}$$

$$E = \frac{hv}{(1 + a)} a$$

as  $(\lambda' - \lambda)$  is not a function of wavelength of incident photon. Thus we can say Compton shift is independent of wavelength of incident photon.

(b) What are polar and non polar molecules? Derive the Clausius-mossotti equation for non-polar solids.

### Ans. Polar and Non-polar molecules :

we know that every atom consists of positive and negative charges in same magnitudes. The positive charge of the nucleus may be supposed to be concentrated at a single point known as centre of gravity of the positive charge. Similarly there will be a centre of gravity of the negative charge. When the two centres of gravity coincide, the molecule is called non-polar molecules. When the two centres of gravity do not coincide, the molecules is called polar molecule.

**Clauius Mossotti equation :** Let us take elemental dielectric having cubic structure. As there are no ions and permanent dipoles in these materials, the ionic polarizability  $\alpha_i$  and orientational polarizability  $\alpha_0$  are zero.

$$\alpha_i = \alpha_0 = 0$$

Thus polarization

$$P = N\alpha_e E_i$$

$$= N\alpha_e \left( E + \frac{P}{3\epsilon_0} \right)$$

$$\text{i.e. } P \left[ 1 - \frac{N\alpha_e}{3\epsilon_0} \right] = N\alpha_e E$$

$$\text{or } P = \frac{N\alpha_e E}{\left( 1 - \frac{N\alpha_e}{3\epsilon_0} \right)} \quad \dots(1)$$

We know that,

$$D = P + \epsilon_0 E$$

$$\text{or } P = D - \epsilon_0 E$$

$$\text{or } \frac{P}{E} = \frac{D}{E} - \epsilon_0$$

$$= \epsilon - \epsilon_0$$

$$[\because D = \epsilon_0 E]$$

$$= \epsilon_0 \epsilon_r - \epsilon_0$$

$$[\because \epsilon_r = \frac{\epsilon}{\epsilon_0}]$$

$$\text{or } P = E\epsilon_0 (\epsilon_r - 1) \quad \dots(2)$$

From equation (1) and (2) we have

$$P = E\epsilon_0 (\epsilon_r - 1) = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}}$$

or

$$1 - \frac{N\alpha_e}{3\epsilon_0} = \frac{N\alpha_e}{\epsilon_0 (\epsilon_r - 1)}$$

or

$$1 = \frac{N\alpha_e}{\epsilon_0 (\epsilon_r - 1)} + \frac{N\alpha_e}{3\epsilon_0}$$

$$= \frac{N\alpha_e}{3\epsilon_0} \left( \frac{3}{\epsilon_r - 1} + 1 \right)$$

$$= \frac{N\alpha_e}{3\epsilon_0} \left( \frac{3 + \epsilon_r - 1}{\epsilon_r - 1} \right)$$

$$= \frac{N\alpha_e}{3\epsilon_0} \left( \frac{\epsilon_r + 2}{\epsilon_r - 1} \right)$$

Hence

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0} \quad \dots(3)$$

Where  $N$  is the number of molecules per unit volume.

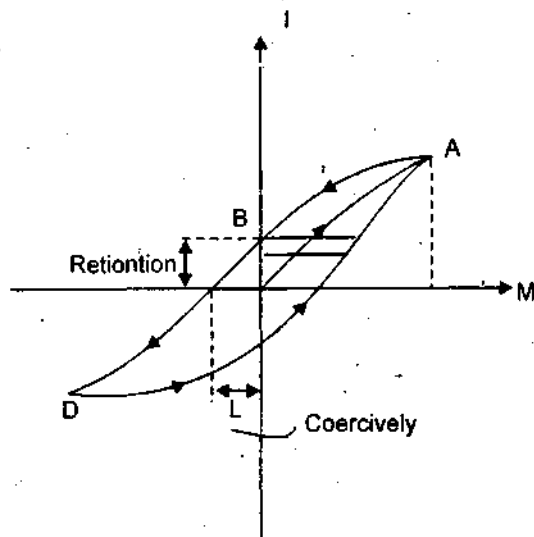
Relation (3) is known as clausius-Mosotti equation. From this relation, we can determine the value of  $\alpha_e$  knowing the value of  $\epsilon_r$ .

**Q.5. Attempt ant one part of the following :**

(a) Draw and explain the curve between magnetic induction  $B$  and magnetic field intensity  $H$  for ferromagnetic material. Also write important application of the curve,.

**Ans. Hysteresis :** The magnetisation curve of a ferromagnetic material is not a linear curve i.e., the variation of magnetic flux  $B$  (or  $\phi$ ) does not vary linearly with the applied field  $H$ . The curve of  $B$  (or  $\phi$ ) versus  $H$  in which the material is magnetised in one direction then in other direction is known as hysteresis curve.

If we vary the magnetising field  $H$ , the intensity of magnetisation  $I$  of the increases along  $OA$  non-uniformly in the substance. At the point  $A$  the material acquires a state of magnetic saturation. Further increase in field does not produce any increase in  $I$ .



If now the magnetising field  $H$  is decreased, the magnetisation  $I$  of material also decreases but does not follow the original path. So  $I$  lags behind  $H$ . When  $H$  becomes zero  $I$  still has a value equal to value 'OB'. This is known as 'Residual magnetism'. The power of retaining this magnetism is called the retentivity of the material so retentivity of the material is a measure of remaining magnetism in the material.

If further now the field is increased in reverse direction the  $I$  decreases along BC, still lagging behind  $H$  until it becomes zero at C i.e., equals to OC. The value of OC is called the coercivity of the substance. So coercivity is a measure of reverse magnetising field required to destroy the magnetism. As  $H$  is increased beyond OC the substance is increasingly magnetised in opposite direction. At point D the substance is again magnetised saturated.

By taking  $H$  back from its maximum -ve value (through zero) to its original value a symmetrical curve is obtained. There are two points at which the substance is magnetised in absence of applied field. This is known as permanent magnetism.

Thus the magnetisation  $I$  and also the magnetic flux density always lag behind the field. This lagging of  $I$  or  $B$  behind  $H$  is called the hysteresis.

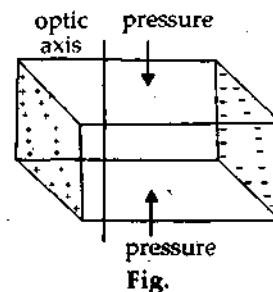
**Important application of the curve :** The hysteresis curve help us to choose the material for making permanent magnets, electromagnet core, transformer cores, Dynamo core, chokes, telephone diaphragm and magnetic shielding.

**(b) How ultrasonic waves are detected ?**  
Write some applications of these waves.

**Ans. Detection of Ultrasonics**

We cannot directly detect the ultrasonic although some animals specially the bat can do so. When the ultrasonics can be detected follows :

**1. Sensitive flame method :** If a narrow sensitive flame is moved in a medium where ultrasonic waves are present, the flame remain stationary at antinodes and flickers at nodes.



**2. Piezo-electric detector :** The quartz crystal can also be used for the detection of ultrasonics. One pair of faces of quartz crystal is subjected to ultrasonics as shown in fig. a. On the other faces which are perpendicular to the previous one, varying electric charges are produced. Of course, the charges are very small. These charges are thus, amplified and then detected by some suitable means.

**3. Thermal detector method :** In this method a fine platinum wire is moved in the medium of ultrasonic waves. The temperature of the medium changes because of alternate compressions and rarefactions. There is a



change of temperature at nodes while at antinodes, the temperature remains constant. Thus, the resistance of the platinum wire changes at nodes and remains constant at antinodes. The change in the resistance of platinum wire with respect to time can be detected from a sensitive bridge arrangement. The bridge will be in the balanced position when the platinum wire is at antinodes.

**4. Kundt's tube method :** A Kundt's tube can be used to detect ultrasonic waves of relatively large wavelength as done for audible sound waves. If ultrasonic waves are passed through the tube, the lycopodium powder sprinkled in the tube collects in the form of heaps at the nodal points and is blown off at the antinodal points.

### Applications of Ultrasonic Waves

**(1) Detection of flaws in metals :** Ultrasonic waves can be used to detect flaws in metal. We know that flaw in the metal gives a change in the medium due to which reflection of ultrasonic waves takes place. Thus when ultrasonic waves pass through a metal having some hole or crack inside it, an appreciable reflection occurs. The reflection also takes place at the back surface of the metal. The reflected pulses are picked up through receiver and are suitably amplified. These pulses are now applied to one set of plates of cathode ray oscillograph. The transmitted signal and reflected signal from the flaw and back surface of metal gives, a peak each. The position of the second peak on the time base of oscillograph will give distance of flaw.

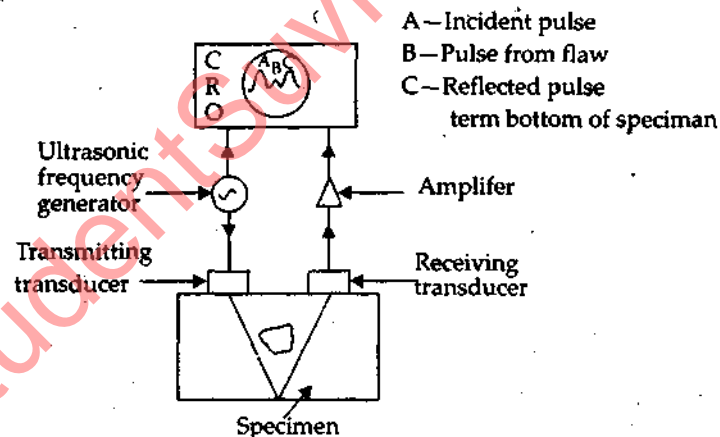


Fig. Ultrasonic flaw detector

The experimental arrangement is given in Fig. Here the transmitting transducer sends a beam of ultrasonics from the material under test. In the presence of flaw in the specimen, the waves will be reflected back and the corresponding recorded intensity in the receiver will be very weak. Similarly, when there is a crack in the specimen, the transmitted waves will have the intensity extremely small. The reflected beam is recorded by using cathode ray oscilloscope.

**(2) Sonar :** It is possible to determine the presence of submerged submarines or an enemy aircraft by a system called Sonar. Sonar is a device which stands for sound Navigation and Ranging. In this system, a sharp ultrasonic beam is directed in different directions into the sea. These are picked up on their return after reflection. The reflection of waves from any direction shows the presence of some reflecting body in the sea. The time interval between the generation of ultrasonic waves and their return after reflection gives the idea of the distance of the body. The change in



frequency of the echo signal due to Doppler effect helps to find the velocity of the body and its direction.

**(3) Soldering and metal cutting :** Ultrasonic waves can be used for drilling and cutting processes in metals. These waves can also be used for soldering, e.g. aluminium cannot be soldered by normal methods. To solder aluminum ultrasonic wave along with electrical soldering iron is used. Ultrasonic welding can be done at room temperatures.

**(4) Depth of sea :** We know that ultrasonic waves are highly energetic and show a little diffraction effect. Hence they can be used for finding the depth of the sea. The time interval between sending the wave and the reflected wave from the sea is recorded. Since the velocity of the waves is known, thus the depth of the sea can be estimated.  $\text{Depth of sea} = vt/2$ .

**(5) Formation of alloys :** The constituents of alloys, having widely different densities, can be kept mixed uniformly by a beam of ultrasonics. Hence it is easy to get alloy of uniform composition.

**(6) Direction signalling :** The ultrasonic waves can be concentrated into a sharp beam because of smaller wavelength and hence can be used for signalling in a particular direction.

**(7) Cleaning and clearing :** These waves can be used for cleaning utensils, washing clothes, removing dust and soot from the chimney.

**(8) Detection of abnormal growth :** Abnormal growth in the brain, certain tumours which cannot be detected by X-rays can be from ultrasonic waves.

**(9) Ultrasonics in metallurgy :** To irradiate molten metals which are in the process of cooling so as to refine the grain size and to protect the formation of cores and to release trapped gases, the ultrasonic waves are used.

**(10) Ultrasonic mixing :** A colloid solution or emulsion of two non-miscible liquids like oil and water can be formed through simultaneously subjecting to ultrasonic radiations. Now-a-days most of the emulsions like polishes, paints, food products and pharmaceutical preparations are prepared by using ultrasonic mixing.

**(11) Treatment of neuralgic pain :** The body parts effected because of neuralgic or rheumatic pains on being exposed to ultrasonics get great relief from pain.

**(12) Coagulation and crystallisation :** The particles of suspended liquid, by ultrasonics, can be brought quite close to each other thus coagulation may take place. The crystallisation rate is also affected by ultrasonics. The size of crystals, if molten metal is put to crystallisation can be made smaller and more uniform by the use of ultrasonics.

**(13) Destruction of lower life :** The animals like rats, frogs, fishes *etc.* can be killed or injured by high intensity ultrasonics.

**Q.6. Attempt any one part of the following :**

**(a) What do you understand by displacement current ? Explain the modification of Ampre's law with displacement current.**

**Ans.**

**Displacement current :**

Maxwell postulated that Displacement current is not only the current in a conductor that produces magnetic field but a changing electric field in vacuum also produces a magnetic field. Hence, a changing electric field is equivalent to a current which flows as long as the electric field in the circuit changes and results the magnetic field. This current is known as displacement current.

### Modification of ampere's Law with displacement current

Ans. (i)  $\nabla \cdot D = 0$  or  $d\nabla \cdot \epsilon = 0$

(ii)  $\nabla \cdot B = 0$  or  $\nabla \cdot H = 0$

(iii)  $\nabla \times E = -\frac{\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t}$

(iv)  $\nabla \times H = \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial \epsilon}{\partial t}$

Now  $\text{curl } H = J + \frac{\partial D}{\partial t}$

From Ampere's circuital law,

$$\oint_C H \cdot dl = I \quad \dots(1)$$

The current  $I$  may be expressed in terms of current density  $J$  as

$$I = \int_S J \cdot ds \quad \dots(2)$$

From (1) & (2),  $\oint_C H \cdot dl = \int_S J \cdot ds$

$$\oint_C \text{curl } H \cdot ds = \int_S J \cdot ds$$

(by using stokes theorem to change line integral into surface integral)

$$\text{curl } H = J \quad \dots(3)$$

Now, divergence of equation  $\text{div } \text{curl } H = \text{div } J$

But  $\text{div } \text{curl } H = 0$ ;

$$\text{div } J = 0 \quad \dots(4)$$

From continuity eq. we have

$$\text{div } J + \frac{\partial \rho}{\partial t} = 0$$

or

$$\text{div } J = -\frac{\partial \rho}{\partial t}$$

$\therefore$  equation (4) to be valid,  $\frac{\partial \rho}{\partial t}$  should be zero, i.e., the charge should be static.

Hence, to include the time varying fields, Maxwell suggested that Ampere's law must be modified. The current density  $J$  should be replaced by  $J + J_d$ ; where  $J_d$  is the current density for displacement current.

the equation (3) becomes.

$$\text{curl } H = J + J_d \quad \dots(5)$$

$\text{div } \text{curl } H = \text{div } J + J_d$  (By taking div)

$$\text{div } J + \text{div } J_d = 0$$

$$\text{div } J_d = -\text{div } J$$

But from continuity eq.;  $\text{div } J = -\frac{\partial P}{\partial t}$ ;

Hence 
$$\text{div } J_d = \frac{\partial P}{\partial t}$$

From Gauss's law in differential form, we have

$$\text{div } D = P \quad \dots(6)$$

Substituting the value of  $P$  from equation (6) we get

$$\text{div } J_d = \frac{\partial}{\partial t} (\text{div } D) = \text{div} \left( \frac{\partial D}{\partial t} \right)$$

This gives 
$$J_d = \frac{\partial D}{\partial t} \quad \dots(7)$$

Substituting the value of  $J_d$  from eq. (7) in (5), we get

$$\text{curl } H = J = \frac{\partial D}{\partial t}$$

Thus, the Maxwell's fourth eq. is the modified form of Ampere's law.

**(b) Write down Maxwell's equations in conducting medium and using these equations derive wave equations for both electric and magnetic fields.**

**Ans.** The charge inside the conductor is zero but it may reside at surface, i.e. the volume charge density  $\rho$  equals zero.

In conducting medium the Maxwell's equations are given by :

$$\nabla \cdot \vec{E} = 0 \quad \dots(i)$$

$$\nabla \cdot \vec{B} = 0 \quad \dots(ii)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad [\because \vec{B} = \mu \vec{H}] \quad \dots(iii)$$

and 
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \dots(iv)$$

$$[\because \vec{J} = \sigma \vec{E} \text{ and } \vec{D} = \epsilon \vec{E}] \sigma = \text{conductivity}$$

On taking curl of equation (ii) we obtain

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

Expanding the vector triple product in left hand side and substituting the value of  $\nabla \times \vec{H}$  from Maxwell's fourth equation, we get 
$$\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} \left[ \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

or 
$$\nabla^2 \vec{E} - \sigma \mu \frac{\partial \vec{E}}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad [\because \nabla \cdot \vec{E} = 0] \quad \dots(v)$$

If we proceed in a similar manner by taking curl of maxwle's fourth equation and then using second and third equation in it, weobtain.

$$\nabla^2 \vec{H} - \sigma \mu \frac{\partial \vec{H}}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad \dots(vi)$$

Equation (v) and (vi) are the wave equations for electromagnetic fields  $\vec{E}$  and  $\vec{H}$  in a homogeneous isotropic conducting medium.

The solutions of equations (v) and (vi) will be of the form  $\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$  and  $\vec{H} = \vec{H}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$

On substituting the value of  $\vec{E}$  in equation (v), we get

$$\nabla^2 [\vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}] - \sigma \mu \frac{\partial [\vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}]}{\partial t} - \epsilon \mu \frac{\partial^2 [\vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}]}{\partial t^2} = 0$$

or 
$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] - \sigma \mu \vec{E} [-i\omega] - \epsilon \mu \vec{E} (-i\omega)(-i\omega) = 0$$

or 
$$\frac{\partial^2}{\partial x^2} [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] + \frac{\partial^2}{\partial y^2} [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] + \frac{\partial^2}{\partial z^2} [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] + i\sigma\mu\omega\vec{E} + \mu\omega^2\vec{E} = 0$$

or 
$$[-(k_x^2 + k_y^2 + k_z^2) + i\sigma\mu\omega + \mu\epsilon\omega^2] \vec{E} = 0$$

or 
$$-k^2 + i\sigma\mu\omega + \mu\epsilon\omega^2 = 0$$

or 
$$k^2 + \mu\epsilon\omega^2 \left( 1 + \frac{i\sigma}{\omega\epsilon} \right) = 0 \quad \dots(vii)$$

Thus the wave propagation vector is in complex from

Let  $k = A + iB$

then  $k^2 = A^2 - B^2 + 2iAB \quad \dots(viii)$

Comparing (vii) and (viii), we obtain,

$$A^2 - B^2 = \mu\epsilon\omega^2 \quad \text{and} \quad 2AB = \mu\omega\sigma$$

Thus

$$A = \omega \sqrt{\mu \epsilon} \left[ \frac{\sqrt{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2} + 1}{2} \right]^{1/2}$$

and

$$B = \omega \sqrt{\mu \epsilon} \left[ \frac{\sqrt{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2} - 1}{2} \right]^{1/2}$$

For a good conductor  $\frac{\sigma}{\omega \epsilon} \gg 1$

$$\therefore A = B = \left( \frac{\mu \omega \sigma}{2} \right)^{1/2}$$

And thus,  $\vec{E} = \vec{E}_0 e^{i[(A + iB)\vec{n} \cdot \vec{r} - \omega t]}$

$$\vec{E}_0 e^{-B\vec{n} \cdot \vec{r}} e^{i(A\vec{n} \cdot \vec{r} - \omega t)}$$

$$\vec{H} = \vec{H}_0 e^{i[(A + iB)\vec{n} \cdot \vec{r} - \omega t]}$$

$$= \vec{H}_0 e^{-B\vec{n} \cdot \vec{r}} e^{i(A\vec{n} \cdot \vec{r} - \omega t)}$$

These equations show that the amplitude for the wave in conducting medium decreases exponentially with increasing distance due to the presence of term  $e^{-B\vec{n} \cdot \vec{r}}$

**Q.7. Attempt any one part of the following :**

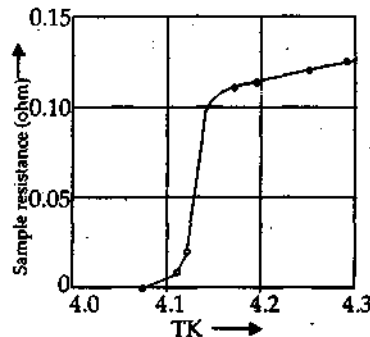
**(a) What are superconductors ? Explain the effect of magnetic field on superconductors.**

**Ans. Super Conductors OR Super conductivity :**

**Ans. Super Conductivity :** The electrical resistance  $R$  of a metal is defined as the ratio of potential difference  $V$  volts applied across the piece of the material to the current  $i$  amp. flowing through it.

i.e.,

$$R = \frac{V}{i} \text{ ohms}$$



**Fig. a : Resistance of mercury as a function of temperature**

Suppose  $l$  be the length of a piece of homogeneous material with uniform cross-sectional area  $A$ , then its specific resistance or resistivity  $\rho$  defined as

$$\rho = R \cdot \frac{A}{l}$$

or

$$R = \rho \frac{l}{A} = \frac{1}{\sigma} \times \frac{l}{A}$$

where  $\sigma = \frac{1}{\rho}$  known as conductivity.

At room temperature, the resistivity for good conductors is of the order of  $10^{-8}$  ohm-m and for insulators, it is between the range of  $10^8$  to  $10^{16}$  ohm-m. Semiconductors have  $\rho$  of the order of  $10^{-1}$  to  $10^1$  ohm-m. Metals have a positive temperature coefficient. Of course, the decrease in resistance is directly proportional to the decrease in temperature. Before the production of low temperatures, it was thought that the electrical resistance of a conductor becomes zero only at absolute zero. But it is observed that the electrical resistivity of many metals and alloys drop to zero when they are cooled to a sufficiently low temperature. e.g., K Onnes in 1911 observed that the electrical resistance of pure mercury suddenly drops to zero as it is cooled below 4.2 K.

**Effect of magnetic field on superconductors :** Effect of external magnetic field : Kammerlingh Onnes found in 1913 that superconductivity vanishes if a sufficiently strong magnetic field is applied. The minimum magnetic field which is necessary to regain the normal resistivity is known as the **critical magnetic field**,  $H_C$ . If the applied magnetic field exceeds the critical value  $H_C$ , the superconducting state is destroyed and the material goes into the normal state. Clearly, the value of  $H_C$  varies with temperature. Fig. shows the dependence of  $H_C$  on temperature in a typical superconductor. At any temperature  $T < T_C$ , the material remains superconducting until a corresponding critical magnetic field is applied. If the magnetic field exceeds the critical value, the material goes into the normal state. The critical field required to destroy the superconducting state decreases progressively with increasing temperature. e.g. a magnetic field of 0.04 T will destroy the superconductivity of mercury at  $T \approx 0$  K, whereas a field of 0.02 T is sufficient to destroy its superconductivity at about 3 K.

The dependence of critical field on temperature is governed as follows:

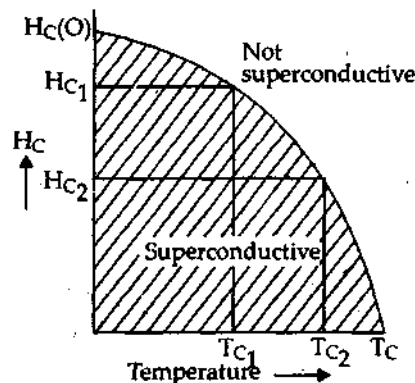


Fig. Schematic representation of the critical magnetic field as a function of temperature



$$H_c(T) = H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right) \right]^2$$

where  $H_c(0)$  is the critical magnetic field at 0K.

**(b) What are carbon nanotubes? Write their important properties and applications.**

**Ans.**

### **Carbon Nanotube**

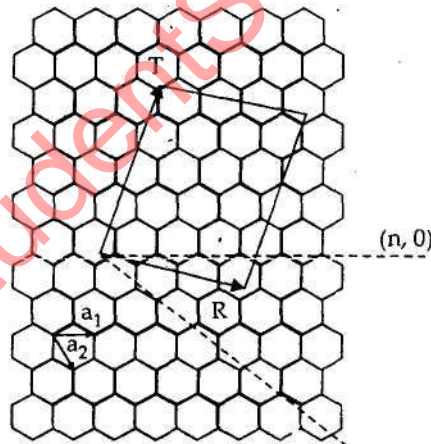
Carbon nanotubes (CNT) are sheets of graphite (graphite is an allotropic<sup>1</sup> form of pure and brittle form of carbon) rolled up to make a tube it means carbon macro-molecules in cylindrical form. The nanotube dimensions are variable and can be as small as 0.4 nm in diameter. A typical computer generated model of carbon nanotubes is give in figure.

### **Properties of Carbon Nanotubes**

- (i) Carbon nanotubes are very hydrophobic and can easily be bind to proteins. Because of this property, they can serve as chemical and biological sensors.
- (ii) They also exhibit interesting electrical properties i.e. depending on the way the graphite structure spirals around the tube, CNTs can be insulating, semiconducting or conducting.
- (iii) CNTs are very light, flexible, thermally stable and durable, and possess extraordinary tensile strength. CNTs have tensile strength of about 65 GPa which is 50 times higher than steel.

### **Structure of Carbon Nanotubes**

The bonding in carbon nanotubes is  $sp^2$ , with each atom joined to three neighbours similar to those in graphite structure. The tubes can thus be considered as rolled-up graphite sheets. The structure of a nanotube can be specified by a **chiral vector**  $(n, m)$  which defines how the graphite sheet is rolled up, where  $n$  and  $m$  are integers of the vectors equation.



**Fig :** The  $(n, m)$  nanotube can be thought of as a vector in an infinite graphite sheet that describes how to "roll up" the graphite sheet to make the nanotube.  $T$  represents the tube axis, and  $a_1$  and  $a_2$  are the unit vectors of graphite.

The chiral vector can be understood with the help of Fig (b). The values of  $n$  and  $m$  determine the *chirality*, or *twist* of the nanotube. The chirality affects the conductance, density, lattice structure and other properties. A single walled CNT is considered metallic if the value is divisible by 3 otherwise, the nanotube is semiconducting. Consequently, if tubes are formed with random values of  $n$  and  $m$ ,



we would expect that two-thirds of nanotubes would be semi-conducting, while the rest would be metallic. Given the chiral vector  $(n, m)$ , the diameter of a carbon nanotube can be determined using the relationship

$$d = (n^2 + m^2 + nm)^{1/2} \cdot 0.0783 \text{ nm}$$

### Types of Carbon Nanotubes

Depending upon the value of  $n$  and  $m$ , the carbon nanotubes have been divided into following three categories :

- (i) If  $n = m$ , the nanotubes are called "armchair".
- (ii) If  $m = 0$ , the nanotubes are called "zigzag".
- (iii) For any other combination of  $n$  and  $m$ , nanotubes are known as "chiral".

### Uses of Carbon Nanotubes

1. The carbon nanotube is often used as a vessel for transporting drugs into the body. The nanotube allows for the drug dosage to hopefully be lowered by localizing its distribution, as well as significantly cut costs to pharmaceutical companies and their consumers.
2. Nanotube based transistors have been made that operate at room temperature.
3. Because of the great mechanical properties of the carbon nanotubule, a variety of structures have been proposed ranging from everyday items like clothes and sports gear to combat jackets.
4. Bulk carbon nanotubes have already been used as composite fibers in polymers to improve the mechanical, thermal and electrical properties of the bulk product.
5. Some other potential applications have been proposed for carbon nanotubes, including conductive and high-strength composites; energy storage and energy conversion devices; sensors; field emission displays and radiation sources; hydrogen storage media; and nanometer-sized semiconductor devices, probes, and interconnect.
6. Carbon nanotubes have also been implemented in nanoelectromechanical systems, including mechanical memory elements and nanoscale electric motors.